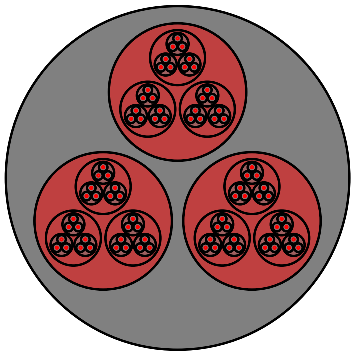

Introduction to Non-Archimedean Geometry

Graduate-level lecture course at IM PAN, Fall 2020

http://achinger.impan.pl/lecture20f.html

Outline

Non-Archimedean or rigid-analytic geometry is an analog of complex analytic geometry over non-Archimedean fields, such as the field of p-adic numbers $\mathbf{Q}_p$ or the field of formal Laurent series $k((t))$. It was introduced and formalized by Tate in the 1960s, whose goal was to understand elliptic curves over a p-adic field by means of a uniformization similar to the familiar description of an elliptic curve over $\mathbf{C}$ as quotient of the complex plane by a lattice. It has since gained status of a foundational tool in algebraic and arithmetic geometry, and several other approaches have been found by Raynaud, Berkovich, and Huber. In recent years, it has become even more prominent with the work of Scholze and Kedlaya in p-adic Hodge theory, as well as the non-Archimedean approach to mirror symmetry proposed by Kontsevich. That said, we still do not know much about rigid-analytic varieties, and many foundational questions remain unanswered.

The goal of this lecture course is to introduce the basic notions of rigid-analytic geometry. We will assume familiarity with schemes.

Course format

(tentative) Two 1hr lectures per week, weekly homework problem sets, no discussion sessions but participation in office hours encouraged. Oral exam.

The course should be available to all graduate students at IM PAN and all students of the University of Warsaw, with appropriate ECTS credit.

Tentative syllabus

0. Outline; topology of p-adic numbers
1. Topological & adic rings
2. Formal schemes I
3. Formal schemes II
4. Tate algebras
5. G-ringed spaces & the admissible topology
6. Rigid-analytic spaces I
7. Rigid-analytic spaces II
8. Examples of rigid-analytic spaces
9. The Tate curve
10. Raynaud's approach
11. Applications
12. Additional topics:
    (a) Huber's theory of adic spaces
    (b) Berkovich spaces
    (c) Riemann–Zariski spaces
    (d) Nagata's compactification theorem

References

[1] Siegfried Bosch. *Lectures on formal and rigid geometry*, volume 2105 of *Lecture Notes in Mathematics*. Springer, Cham, 2014.

[2] Brian Conrad. Several approaches to non-Archimedean geometry. In *p-adic geometry*, volume 45 of *Univ. Lecture Ser.*, pages 9–63. Amer. Math. Soc., Providence, RI, 2008.

[3] Jean Fresnel and Marius van der Put. *Rigid analytic geometry and its applications*, volume 218 of *Progress in Mathematics*. Birkhäuser Boston, Inc., Boston, MA, 2004.

[4] Kazuhiro Fujiwara and Fumiharu Kato. *Foundations of rigid geometry. I.* EMS Monographs in Mathematics. European Mathematical Society (EMS), Zürich, 2018.

[5] John Tate. Rigid analytic spaces. *Invent. Math.*, 12:257–289, 1971.

Questions?

Contact Piotr Achinger `pachinger@impan.pl`