

# PROGRAM FOR THE WORKSHOP “ANALYTIC DE RHAM STACKS”

IMPAN, WARSAW, DECEMBER 9–13, 2024

**General instructions for speakers.** Prioritize presenting the statements and making links to classical theory, e.g., Huber’s adic spaces, but include (ideas of) proofs as much as time permits. Try to not get too hung up on condensed technicalities, or on animated or  $\mathbb{E}_\infty$ -rings, assume the basics of condensed mathematics or animated rings as generally known; could decide to review some inputs as needed, if this clarifies the statements of the results being presented.

Here is the plan for the talks. The references are to [arXiv:2401.07738](https://arxiv.org/abs/2401.07738) or to the bibliography there.

- 1. Adic spaces as categorified locales (Mathew, 1h).** Cover Sections 2.2 and 2.3 quite faithfully, in particular, define categorified locales and explain how Tate adic spaces may be viewed as such. Explain how to do geometry with categorified locales, ideally, giving instructive examples to make this material less dry.
- 2.  $\dagger$ -nil-radical and bounded affinoid rings (Reinecke, 2×1h).** Cover Section 2.6, incorporating some preliminaries from Sections 2.3 and 2.4 as needed. Especially, present Definition 2.6.1, Lemma 2.6.5, Definition 2.6.6, Proposition 2.6.9. The second talk could begin with Definition 2.6.10 and cover material up to the end of the section, especially, Lemma 2.6.18, Corollary 2.6.21.
- 3. Derived Tate adic spaces (Anschütz, 2×1h).** Cover Section 2.7 quite faithfully, in particular, construct the adic spectrum of a bounded affinoid ring, prove some of its fundamental properties, define derived Tate adic spaces.
- 4. Abstract six functor formalisms (Mann, 2×1h).** Cover Section 3.1. Introduce three functor and six functor formalisms, Definition 3.1.5, the notion of a  $\mathcal{D}$ -cover. Introduce the Lu–Zheng category, Definition 3.1.10, mention / expand on Remark 3.1.9, introduce  $f$ -smooth and  $f$ -proper morphisms, discuss their properties. The second talk could start with Section 3.1.3, cover Definitions 3.1.17–3.1.19, 3.1.25, discuss the properties of these notions, ideally, with some examples from classical settings.
- 5. Algebraic topology from the six functor point of view (Hesselholt, 1h).** Overview how classical results in algebraic topology and, perhaps, also algebraic geometry, could be viewed in terms of six functor formalisms.
- 6. Tate stacks (Bosco, 1h).** Cover Sections 3.2 and 3.3. Build up to Definition 3.2.7 (solid  $\mathcal{D}$ -stacks) and then to Definition 3.2.10 (Tate stacks), discuss morphisms of finite presentation.
- 7. The cotangent complex of analytic rings (Koshikawa, 1h).** Cover Sections 3.4 and 3.5. Build the cotangent complex in the analytic ring context with some examples relating to basic properties and more classical situations (Example 3.4.10), especially, present Corollary 3.4.8, Definition 3.4.11. Introduce solid smooth, solid étale maps (Definitions 3.5.4, 3.5.5), discuss their properties as much as possible (Section 3.5), for instance, Corollary 3.5.12.

8. **Derived rigid geometry (Imai, 2×1h)**. Present some elements of geometry of derived Tate adic spaces following Sections 3.6, 3.7. For instance, it would be nice to build up to and prove Lemma 3.6.12, Proposition 3.6.13, state the Serre duality Theorem 3.6.15, perhaps overview its proof to some extent if time permits (an honest proof would require inputs from Section 4). Discuss  $\dagger$ -formally smooth / étale maps and their properties as in Section 3.7.
9. **Cartier duality of vector bundles (Zhang, 2×1h)**. Present Cartier duality for vector bundles following Section 4. It may be natural to focus on the algebraic case (Section 4.2) in the first talk, building to Theorem 4.2.7, then turn to the analytic variant (Section 4.3) in the second talk, building to Theorem 4.3.13.
10. **Algebraic de Rham stack (Ito, 1h)**. Present the “algebraic” theory of the de Rham stack in rigid geometry following Section 5.1. In particular, introduce Definition 5.1.1 (de Rham, Hodge prestacks, algebraic de Rham prestack), discuss Proposition 5.1.4, introduce the six functor formalism of algebraic  $D$ -modules, build up to and present Theorems 5.1.12 and 5.1.13.
11. **Analytic de Rham stack (Le Bras, 1h)**. Present the “analytic” theory of the de Rham stack in rigid geometry following Section 5.2. Discuss the Kashiwara equivalence (Corollary 5.2.5), introduce the six functor formalism of analytic  $D$ -modules, present Theorem 5.2.10, Corollary 5.2.13, discuss the comparison with “algebraic”  $D$ -modules (Section 5.2.3).
12. **Poincaré duality for D-modules (Zavvalov, 1h)**. Present the Poincaré duality theorems for algebraic and analytic  $D$ -modules, expanding on the discussion of Section 5.3.
13. **Locally analytic representations (Dospinescu, 1h)**. Cover Section 6, relatedly, §6 of the introduction; especially, introduce  $G^{\text{sm}}$  and  $G^{\text{la}}$  (Definition 6.2.1), present Proposition 6.2.3. Explain [RJRC23, Theorems 4.3.3 and Proposition 5.4.2].