Problem Set 7

due Dec 13, 2020

Problem 1. Show that finite maps between rigid-analytic spaces are proper.

Problem 2. Let X be a proper scheme over K. Prove that the associated rigid-analytic space X^{an} is proper.

Hint: Use Chow's lemma to show that X^{an} is quasi-compact.

Problem 3. Let $q \in K$ be such that $0 < |q| < 1$, and consider the action of the cyclic group $q^{\mathbf{Z}}$ on the punctured plane $X = (\mathbf{A}^2_K \setminus 0)^{\mathrm{an}}$ by rescaling. Show that the action is properly discontinuous and describe the quotient $Y = X/q^{\mathbf{Z}}$ (called the non-Archimedean Hopf surface) as a union of four affinoids glued along affinoid subdomains.

Problem 4. Let $Y = (\mathbf{A}^2_K \setminus 0)^{\mathrm{an}}/q^{\mathbf{Z}}$ be the non-Archimedean Hopf surface. Compute $H^1(Y, \mathscr{O}_Y)$ and $H^0(Y, \Omega^1_{Y/K})$.

Hint: $H^0(Y, \Omega^1_{Y/K})$ are just the $q^{\mathbf{Z}}$-invariant differentials on X.

Problem 5. Let $K \subseteq K'$ be an extension of non-Archimedean fields (that is, the norm on K' restricts to the norm on K). Define a base change functor

$$(-)_{K'} : \mathrm{Rig}_K \to \mathrm{Rig}_{K'}.$$

Hint: Define the functor first on affinoid algebras, and then pin down $(-)_{K'}$ by a universal property.