Problem Set 10, part II

due Jan 31, 2021

Problem 4. Let $U = \mathbf{A}^{1,\mathrm{an}}_K$, $X = \mathbf{P}^{1,\mathrm{an}}_K$, and let $j\colon U \to X$ be the open immersion. Prove that there does not exist a formal model $\mathfrak{U} \to \mathfrak{X}$ of j which is an open immersion.

Hint: The morphism j is not quasi-compact.

Problem 5. Construct a formal model of the open unit disc $\mathbf{D}^{\circ}_K = \{|x| < 1\} \subseteq \mathbf{D}^1_K$ over $K = k((t))$. What does the special fiber look like?

Hint: Use the finite type covering

$$X = U_0 \cup \bigcup_{n>0} U_n$$

by

$$U_0 = \{|x| \leq |t|\}$$

and

$$U_n = \{|t|^{\frac{1}{n}} \leq |x| \leq |t|^{\frac{1}{n+1}}\} \quad (n > 1).$$