Term paper topics

Due date: Jan 31, 2021

Guidelines[1]

Each student should choose a topic based on their own interests, but all topics should be cleared with me in advance. Some possible topics, some more advanced than others, are suggested below. I am happy to help students choose a topic through individual consultations.

Format. Your paper should be written using Latex, and should be about 5–10 pages in length.

Style. Write your paper in the style of a research paper. That means that it has an introduction which explains the main result, and a clearly organized overall structure. The paper should be centered around one or two main results (as opposed to being a sequence of propositions and lemmas with no clear start or end).

Additional Requirement. Every paper must contain at least one nontrivial example worked out in detail.

Suggested topics

Project 1 (Berkovich spaces)**.** Define the Berkovich space associated to an affinoid K-algebra A (only as a topological space). Explain its relationship with the Riemann–Zariski space of A.

Suggested reading: [14, §2], [8], [21], [12, Appendix II B.3], [18]

Project 2 (Gluing vector bundles)**.** Explain the results of Beauville and Laszlo on formal gluing of vector bundles on an affine scheme, and the generalization of this result to non-affine schemes due to Ben-Bassat and Temkin.

Suggested reading: [2], [3]

Project 3 (Adic spaces)**.** Explain the basics of Huber's theory of adic spaces.

Suggested reading: [6], [13, §1], [16, §2], [12, Appendix II A]

Project 4 (Nagata's compactification theorem)**.** Give an outline of the proof of Nagata's compactification theorem using Riemann–Zariski spaces.

Suggested reading: [12, Appendix II E], [17]

Project 5 (Mumford curves)**.** Outline Mumford's construction of uniformization of curves with totally degenerate reduction.

Suggested reading: [5, Chapitre 6], [15, §2]

[1] Adapted from *https://math.berkeley.edu/~molsson/Termpaper.pdf*

Project 6 (Raynaud uniformization)**.** Explain Raynaud's uniformization of abelian varieties with semiabelian reduction.

Suggested reading: [1], [15, §6]

Project 7 (Perfectoid spaces)**.** Define perfectoid spaces over a perfectoid field and state the tilting equivalence.

Suggested reading: [16], [11], [4]

Project 8 (Drinfeld's upper half-plane)**.** Define Drinfeld's upper half plane and construct its formal model.

Suggested reading: [5, Chapitre 7]

Project 9 (Non-Archimedean Hopf surface)**.** Explain the construction of the non-Archimedean Hopf surface following Voskuil, and construct its formal models.

Suggested reading: [20]

Project 10 (Étale cohomology)**.** Study the basics of étale cohomology of rigid-analytic varieties.

Suggested reading: [7], [13, §0], [9]

Project 11 (Toric Riemann–Zariski spaces)**.** Define and study the space of valuative submonoids of $\mathbf{Z}^r$, as a "tropical" or "toric" analog of the Riemann–Zariski space of $\mathbf{G}_m^r$.

Suggested reading: [10]

Project 12 (Resolution of surface singularities)**.** Study Zariski's original proof of resolution of singularities for surfaces using valuations.

Suggested reading: [22], [19]

References

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[3] Oren Ben-Bassat and Michael Temkin. Berkovich spaces and tubular descent. *Adv. Math.*, 234:217–238, 2013.

[4] Bhargav Bhatt. What is ... a perfectoid space? *Notices Amer. Math. Soc.*, 61(9):1082–1084, 2014.

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[6] Brian Conrad. A brief introduction to adic spaces. `http://math.stanford.edu/~conrad/papers/Adicnotes.pdf`.

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[8] Antoine Ducros. Espaces analytiques p-adiques au sens de Berkovich. Number 311, pages Exp. No. 958, viii, 137–176. 2007. Séminaire Bourbaki. Vol. 2005/2006.

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[13] Roland Huber. *Étale cohomology of rigid analytic varieties and adic spaces*. Aspects of Mathematics, E30. Friedr. Vieweg & Sohn, Braunschweig, 1996.

[14] Kiran S. Kedlaya and Ruochuan Liu. Relative p-adic Hodge theory: foundations. *Astérisque*, (371):239, 2015.

[15] Werner Lütkebohmert. *Rigid geometry of curves and their Jacobians*, volume 61 of *Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics].* Springer, Cham, 2016.

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[18] Michael Temkin. Introduction to Berkovich analytic spaces. In *Berkovich spaces and applications*, volume 2119 of *Lecture Notes in Math.*, pages 3–66. Springer, Cham, 2015.

[19] Michel Vaquié. Valuations and local uniformization. In *Singularity theory and its applications*, volume 43 of *Adv. Stud. Pure Math.*, pages 477–527. Math. Soc. Japan, Tokyo, 2006.

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