Cohomology of Varieties

Mar 28–Apr 1, 2022 at IMPAN

Monday

10:00–11:00	**Gabriel Dospinescu**	*(Non) Finiteness properties of the mod p cohomology of the Drinfeld tower*
	coffee break	
11:30–12:30	**Pierre Colmez**	*On the p-adic pro-étale cohomology of analytic varieties*
	lunch break	
15:00–16:00	**Bogdan Zavyalov**	*Mod-p Poincare Duality in p-adic Analytic Geometry*
	coffee break	
16:30–17:30	**David Hansen**	*Categorical local Langlands conjecture: refinements, questions, and speculations (on Zoom)*

Tuesday

10:00–11:00	**Joseph Ayoub**	*On the classicality of the motivic Galois group*
	coffee break	
11:30–12:30	**Elden Elmanto**	*On the motivic cohomology of singular schemes*
12:45–13:45	**Javier Fresán**	*E-functions and geometry*
	lunch break	
15:30–16:30	**Bruno Klingler**	*Recent progress on Hodge loci*
	coffee break	
17:00–18:00	**Alberto Vezzani**	*On the p-adic weight-monodromy conjecture for complete intersections in toric varieties*

Wednesday

10:00–11:00	**Kęstutis Česnavičius**	*The Bass–Quillen phenomenon for reductive group torsors*
	coffee break	
11:30–12:30	**Quentin Guignard**	*Ramification in higher dimension*
12:45–13:45	**Dmitry Vaintrob**	*Logarithmic Hodge theory via noncommutative geometry and almost mathematics*
	free afternoon	

Thursday

10:00–11:00	**Hélène Esnault**	*Arithmetic properties of strongly rigid local systems*
	coffee break	
11:30–12:30	**Adrian Langer**	*On some representations of fundamental groups of algebraic schemes*
	lunch break	
15:00–16:00	**Peter Jossen**	*On the cohomology of the affine line*
	coffee break	
16:30–17:30	**Masha Vlasenko**	*Cohomology and congruences*
	Dinner at BIBENDA (Nowogrodzka 10) at 19:00	

Friday

9:00–10:00	**Katharina Hübner**	*Logarithmic differentials on discretely ringed adic spaces*
	coffee break	
10:30–11:30	**Remy van Dobben de Bruyn**	*Equivalence of several conjectures on independence of ℓ*
11:45–12:45	**Marcin Lara**	*Geometric arcs and fundamental groups of rigid spaces*
	lunch break	
14:00–15:00	**Antoine Ducros**	*Comparison between étale and topological cohomology of Berkovich spaces*

Lecture room: **321 IMPAN** Coffee breaks: **409 IMPAN** Banach Center office (reimbursements): **426 IMPAN**

Abstracts

Joseph Ayoub • *On the classicality of the motivic Galois group*

The motivic Galois group is most naturally considered as an object in spectral algebraic geometry. However, deep conjectures in the theory of motives imply that the motivic Galois group is classical, i.e., has no higher derived information. We will discuss some recent attempts to verify the classicality of the motivic Galois group.

Kęstutis Česnavičius • *The Bass–Quillen phenomenon for reductive group torsors*

For a regular ring R, the Bass–Quillen conjecture predicts that every vector bundle on the relative affine space $\mathbf{A}^d_R$ descends to R. I will discuss the generalization of this conjecture to torsors under more general reductive groups.

Pierre Colmez • *On the p-adic pro-étale cohomology of analytic varieties*

I will report on the joint work with W. Nizioł and S. Gilles on the p-adic pro-étale cohomology of analytic varieties (comparison theorems, duality, etc.).

Remy van Dobben de Bruyn • *Equivalence of several conjectures on independence of ℓ*

The ℓ-adic étale cohomology of a variety is defined in terms of the geometry of ℓ-power degree coverings. Unlike in algebraic topology, there is no integer-valued theory to compare with, leading to a plethora of "independence of ℓ" conjectures for varieties over finite fields. We will survey what is known, and prove that independence of ℓ for Betti numbers is equivalent to independence of ℓ for homological equivalence of algebraic cycles.

Gabriel Dospinescu • *(Non) Finiteness properties of the mod p cohomology of the Drinfeld tower*

Previous work joint with Colmez and Niziol shows that the p-adic étale cohomology of the Drinfeld tower for $\mathrm{GL}_2(\mathbf{Q}_p)$ has strong finiteness properties. I will discuss a mod p version of such properties, and a strange dichotomy appearing when we move from $\mathrm{GL}_2(\mathbf{Q}_p)$ to $\mathrm{GL}_2(F)$, with F a nontrivial finite extension of $\mathbf{Q}_p$.

Antoine Ducros • *Comparison between étale and topological cohomology of Berkovich spaces*

In algebraic geometry, the "difference" between étale and topological (i.e. Zariski) cohomology is well understood and described through the Bloch–Ogus complex. I will give a survey talk about the analogous problem in the Berkovich setting and discuss what is known as well as some open (and hopefully interesting) questions.

Elden Elmanto • *On the motivic cohomology of singular schemes*

I will report on joint work with Matthew Morrow. Using ideas from topological cyclic homology and p-adic Hodge theory, we constructed a theory of Zariski p-adic motivic complexes for any qcqs scheme in characteristic p. This theory is the associated graded pieces of a motivic filtration on algebraic K-theory and hence form the E_2 page of an extension of the motivic spectral sequence of smooth varieties. A key result is an agreement of this construction with Bloch cycle complexes on smooth varieties which, time permitting, I will explain a proof of.

Hélène Esnault • *Arithmetic properties of strongly rigid local systems*

(joint with Michael Groechenig) We explain what to predict for arithmetic local systems and show, for the strongly rigid ones, that they underlie over p-adic varieties for p large the structure of a Fontaine–Laffaille module.

Javier Fresán • *E-functions and geometry*

(joint work with Peter Jossen) Siegel introduced the notion of E-function in a landmark 1929 paper with the goal of generalising the Hermite–Lindemann–Weierstrass theorem on the transcendence of the values of the exponential function at algebraic numbers. E-functions are power series with algebraic coefficients that are solutions of a linear differential equation and satisfy some growth conditions of arithmetic nature. Last year, Peter and I answered in the negative Siegel's question whether all E-functions could be expressed as polynomials in hypergeometric

functions. In this talk, I will try to amend Siegel's question by explaining how E-functions arise from geometry in the form of "exponential period functions" and why it might seem reasonable, in the light of other conjectures, to expect that all E-functions are of this kind.

Quentin Guignard • *Ramification in higher dimension*

I will discuss a generalization of the (logarithmic) Abbes–Saito ramification theory to general bases (instead of a henselian trait), with a particular focus on the corresponding generalization of Deligne's description of extensions of local fields with bounded ramification.

David Hansen • *Categorical local Langlands conjecture: refinements, questions, and speculations*

At the very end of their epic manuscript, Fargues and Scholze proposed a categorical upgrade of the local Langlands correspondence, relating coherent sheaves on the stack of L-parameters with ℓ-adic sheaves on the stack of G-bundles over the Fargues–Fontaine curve. Much like the classical local Langlands conjecture, their proposal is not completely precise. The main theme of this talk is that meditating on this categorical local Langlands conjecture, and trying to make it precise, is a very good way of generating additional conjectures.

Katharina Hübner • *Logarithmic differentials on discretely ringed adic spaces*

The object of interest in this talk is a certain subsheaf Ω_X^+ of the sheaf of differentials Ω_X of a discretely ringed adic space X over a field k defining an integral structure on Ω_X. We will define Ω_X^+ using Kähler seminorms and establish a relation with logarithmic differentials. Finally we study the case where $X = \mathrm{Spa}(U, Y)$ for a scheme Y over k and a subscheme U such that the corresponding log structure on Y is log smooth. It turns out that $\Omega_X^+(X)$ equals $\Omega^{\log}_{(U,Y)}(U, Y)$.

Peter Jossen • *On the cohomology of the affine line*

(joint with Javier Fresán). Consider a ramified cover of the affine line by itself, given by some nonconstant polynomial. The direct image of the constant sheaf, appropriately shifted and modulo constants, is a perverse sheaf on the affine line with trivial cohomology. Perverse sheaves of this type are fairly easy to describe in elementary terms. They form a tannakian category with respect to convolution, and have been studied (e.g. by Katz) in connection with exponential sums. In my talk, I will explain what we know and don't know about the associated tannakian Galois group.

Bruno Klingler • *Recent progress on Hodge loci*

Given a smooth family of projective varieties on a quasi-projective base S, its Hodge locus is the set of closed points of S where the fiber admits "exceptional" Hodge classes. In 1995 Cattani, Deligne and Kaplan showed that it is a countable union of algebraic subvarieties of S, as predicted by the Hodge conjecture. In this talk I will discuss the recent progress in understanding the geometry and arithmetic of the Hodge locus: in "most cases" it is actually algebraic (rather than a countable union of algebraic subvarieties of S), defined over a number field if the family is. Based on works of Baldi–Klingler–Ullmo, Klingler–Otwinowska–Urbanik, Kreutz.

Adrian Langer • *On some representations of fundamental groups of algebraic schemes*

Let X be a smooth projective variety with some fixed polarization H. It is known that a strongly slope H-semistable vector bundle E with vanishing rational Chern classes is numerically flat (i.e., both E and its dual are nef). Over complex base field such bundles arise from unitary representations of the topological fundamental group. The talk will be devoted to various generalizations and characterizations of this class of bundles and its applications to geometry of algebraic varieties.

Marcin Lara • *Geometric arcs and fundamental groups of rigid spaces*

We introduce a new category of coverings in rigid geometry, called geometric coverings, and show it is classified by a certain topological fundamental group. Geometric coverings generalize the class of étale coverings, introduced by de Jong, and its various natural modifications, and have certain desirable properties that were missing from those older notions: they are étale local and closed under taking infinite disjoint unions. The definition is based on the property of unique

lifting of "geometric arcs." On the way, we answer some questions from the foundational paper of de Jong. This is joint work with Piotr Achinger and Alex Youcis.

Alberto Vezzani • *On the p-adic weight-monodromy conjecture for complete intersections in toric varieties*

We give a proof of the p-adic weight monodromy conjecture for scheme-theoretic complete intersections in proper smooth toric varieties. The strategy is based on Scholze's proof in the ℓ-adic setting, that can be adapted using recent homotopical results developed in the context of rigid analytic motives. Joint work with F. Binda and H. Kato.

Dmitry Vaintrob • *Logarithmic Hodge theory via noncommutative geometry and almost mathematics*

I will give a definition of a certain category of "log quasicoherent" sheaves on a logarithmic variety which uses Faltings's "almost mathematics" and which has the property that log differential forms and log polyvector fields are the Hochschild homology (appropriately understood) and Hochschild cohomology, respectively, of this category. This implies a certain "noncommutative Hodge theory" associated to a log variety in mixed characteristic. I will also explain (if there is time left over) a relationship of the proof of the main results to mirror symmetry.

Masha Vlasenko • *Cohomology and congruences*

We consider the module of differential forms on the complement of a toric hypersurface. For a positive integer k we define the submodule of so-called *kth formal derivatives*, which can be characterized by certain divisibility properties of their expansion coefficients. It turns out that under a certain condition, which we call the *kth Hasse-Witt condition*, the quotient of all differential forms by the *kth formal derivatives* is a free module of finite rank. This result is a p-adic phenomenon, and for $k = 1$ we recover an analog of the explicit construction of unit-root crystals given by Nickolas Katz in the 1980s. We will also discuss several applications of our result. For $k = 1$ one obtains Dwork's congruences and Gauss' congruences for expansion coefficients of rational functions. With higher k our theorem can be used to explain some *supercongruences*, to show existence and derive some p-adic properties of the excellent *Frobenius lifts* and to prove integrality of the mirror map and integrality of instanton numbers in some key examples of mirror symmetry. This is joint work with Frits Beukers.

Bogdan Zavyalov • *Mod-p Poincaré duality in p-adic analytic geometry*

Étale cohomology of $\mathbf{F}_p$-local systems does not behave nicely on general smooth p-adic rigid-analytic spaces; e.g., the $\mathbf{F}_p$-cohomology of the 1-dimensional closed unit ball is infinite.

However, it turns out that the situation is much better if one considers only proper rigid-analytic spaces. These spaces have finite $\mathbf{F}_p$ cohomology groups and these groups satisfy Poincaré duality if X is smooth and proper.

I will explain how one can prove such results using the concept of almost coherent sheaves that allows one to "localize" such questions in an appropriate sense and actually reduce to some local computations.

I will also mention some generalizations of these results in the relative setting (in progress).