Wild Ramification and Irregular Singularities

Sep 23–27, 2019 at IMPAN

Lecture room: **321 IMPAN**
Coffee breaks: **409 IMPAN**
Banach Center office (reimbursements): **426 IMPAN**

Monday

9:00–10:00	**Takuro Mochizuki**	*Riemann–Hilbert correspondence and Fourier transform (I)*
	coffee break	
10:30–11:30	**Jean-Baptiste Teyssier**	*Characteristic cycle and wild ramification for the nearby cycles of étale sheaves*
11:45–12:45	**Vaidehee Thatte**	*Ramification theory for arbitrary valuation rings in positive residue characteristic*
	lunch break	
15:00–16:00	**Kay Rülling**	*Reciprocity sheaves and abelian ramification theory*
	coffee break	
16:30–17:30	**Peter Jossen**	*E-functions and families of exponential motives*

Tuesday

9:00–10:35	**Alexander Beilinson**	SS
	coffee break	
11:00–12:00	**Michael Temkin**	*Reduction and lifting problems on Berkovich curves*
	lunch break	
14:30–15:30	**Tomoyuki Abe**	*Characteristic cycles and higher homotopies (I)*
	coffee break	
16:30–17:30	**Daniel Litt**	*Finiteness and discreteness results for representations of arithmetic fundamental groups (I)*

Wednesday

9:00–10:35	**Takeshi Saito**	CC
	coffee break	
11:00–12:00	**Jérôme Poineau**	*De Rham cohomology of p-adic analytic curves*
12:10–13:10	**Enlin Yang**	*On the semi-continuity of characteristic cycles for étale sheaves*
	free afternoon	

Thursday

9:00–10:00	**Takuro Mochizuki**	*Riemann–Hilbert correspondence and Fourier transform (II)*
	coffee break	
10:30–11:30	**Moritz Kerz**	*Étale local systems of rank one*
11:45–12:45	**Daniel Litt**	*Finiteness and discreteness results for representations of arithmetic fundamental groups (II)*
	lunch break	
15:00–16:00	**Tomoyuki Abe**	*Characteristic cycles and higher homotopies (II)*
	coffee break	
16:30–17:30	**Anna Cadoret**	*Ultraproduct Weil II for curves and $\mathbf{Z}_\ell$-companions*

Friday

9:00–10:00	**Adrian Langer**	*Nearby-cycles and semipositivity in positive characteristic*
	coffee break	
10:30–11:30	**Tony Pantev**	*Symplectic structures on moduli of Stokes data*
11:30–12:30	**Claude Sabbah**	*What are irregular perverse sheaves?*

Abstracts

Tomoyuki Abe • *Characteristic cycles and higher homotopies (II)*

Saito introduced functions with isolated characteristic points in his celebrated work on characteristic cycles. If one fixes a variety, a key of Saito's construction is that we have sufficient supply of such functions for defining characteristics cycles, using Beilinson's definition of characteristic variety. However, we also face a situation where we can't expect for sufficient supply of such functions. My goal (unfortunately still in progress) is to deal with such a bad situation. For doing this, recent progress of the theory of higher dimensional nearby cycle, after Orgogozo, is crucial. I'll try to review this theory in the first lecture, and report on my work in progress with D. Patel in the second half.

Alexander Beilinson • SS

I will explain main results of the theory of singular support of étale sheaves and sketch the proofs.

Anna Cadoret • *Ultraproduct Weil II for curves and $\mathbf{Z}_\ell$-companions.*

I will briefly explain how to define a category of lisse sheaves with ultraproduct coefficients and construct, in this setting, a partial formalism of Frobenius weights where, at least, the corresponding version of the fundamental theorem of Weil II for curves is available. Combining this result with geometric arguments, one can deduce (without restriction on the dimension of the variety) most of the classical corollaries of Weil II (purity, geometric semisimplicity, Tannakian Cebotarev etc.). This ultraproduct formalism has applications to integral models in compatible systems of lisse ℓ-adic sheaves: asymptotic unicity of integral models, asymptotic geometric semisimplicity of the reduction modulo-ℓ, generalization of Gabber's torsion freeness theorem for higher direct images etc. From these results, one also deduce the Langlands correspondance for ultraproduct coefficients and an asymptotic form of a Langlands correspondence modulo-ℓ, which implies the asymptotic existence and unicity of the lift in de Jong's conjecture on modulo-ℓ representation of the étale fundamental group.

Peter Jossen • *E-functions and families of exponential motives*

We present a few concrete examples of families of exponential motives on the affine line over a number field, and observe that some of the associated period functions are "arithmetic Gevrey series," e.g. E-functions. These period functions are solutions of a Gauss–Manin differential equation, which is typically irregular singular at infinity. We explain how one may compute the differential Galois group. A transcendence theorem of Siegel and Shidlovskii relates the dimension of this Galois group with special values of the E-functions, i.e. periods of a particular exponential motive.

Moritz Kerz • *Étale local systems of rank one*

In joint work with H. Esnault we study deformation spaces of rank one ℓ-adic local systems on smooth projective varieties. We generalize some results classically shown using complex Hodge theory to positive characteristic.

Adrian Langer • *Nearby-cycles and semipositivity in positive characteristic*

I will talk about an analogue of Hodge theory in positive characteristic and its applications. In particular, I wil prove some strong semipositivity theorems for analogs of complex polarized variations of Hodge structures. This implies semipositivity for the relative canonical divisor of a semistable reduction and it also gives some new results over complex numbers.

Daniel Litt • *Finiteness and discreteness results for representations of arithmetic fundamental groups*

In part I, I will discuss (following Esnault and Kerz), the proof of Deligne's finiteness results for representations of arithmetic fundamental groups of curves over finite fields with bounded wild ramification, via sphere packing, and a corollary of this result for representations of geometric fundamental groups into $\mathrm{GL}_n(\mathbf{Q}_p)$.

In part II, I will discuss the dynamics of the Galois action on the space of representations of the geometric fundamental group of a curve over a finitely-generated field, and implications for e.g. level structures on Abelian schemes over curves.

Takuro Mochizuki • *Riemann–Hilbert correspondence and Fourier transform*

According to the classical Riemann–Hilbert correspondence, meromorphic flat bundles on curves are classified by local systems with Stokes structure, or more roughly, tuples of vector spaces and linear maps. It is natural to ask how a transformation for meromorphic flat bundles is explicitly computed in terms of the corresponding topological objects or linear algebraic objects.

In the first talk, we plan to review the classical Riemann–Hilbert correspondence in the one dimensional case, and some well known results on the Fourier transform. In the second talk, we would like to explain our rather recent study on the basis of the notion of Stokes shells, which is another way to describe Stokes structure.

Tony Pantev • *Symplectic structures on moduli of Stokes data*

I will discuss the notion of shifted symplectic structures along the stalks of constructible sheaves of derived stacks on stratified spaces. I will describe a general pushforward theorem producing relative symplectic forms and will explain explicit techniques for computing such forms. As an application I will describe a universal construction of Poisson structures on derived moduli of local systems on smooth varieties and will explain how symplectic leaves arise from fixing irregular types and local formal monodromies at infinity. This is a joint work with Dima Arinkin and Bertrand Toën.

Jérôme Poineau • *De Rham cohomology of p-adic analytic curves*

Consider a smooth p-adic analytic curve and a module with an integrable connection on it. Following Francesco Baldassarri, we will explain how to attach a radius of convergence to any point of the curve. The radius map turns out to have a simple behavior: it factors through the retraction to a locally finite subgraph of the curve. Using this strong property, we are able to prove a criterion for finite dimensionality of the de Rham cohomology of the connection and an index formula.

This is joint work with Andrea Pulita.

Kay Rülling • *Reciprocity sheaves and abelian ramification theory*

Reciprocity sheaves were introduced by Kahn–Saito–Yamazaki as a generalization of homotopy invariant sheaves with transfers. For example, all smooth commutative group schemes over the perfect base field k define a reciprocity sheaf. I will report on joint work with Shuji Saito. For any reciprocity sheaf $\mathcal{F}$ we associate a motivic conductor which corresponds to a cycle-theoretic defined filtration on $\mathcal{F}(L)$, for L a henselian discrete valuation field of geometric type over k (hdvf). In particular, for an element $f \in \mathcal{F}(U)$, where U is smooth over k with proper model X such that $X \setminus U$ is the support of a Cartier divisor, the motivic conductor of f at all hdvf's determines a divisor at infinity on X which can be viewed as a measure for the ramification or the pole order of f at infinity. In general we show that a reciprocity sheaf is the same as a Nisnevich sheaf with transfers which has such a conductor. Furthermore, given a number n we give a criterion when the ramification divisor of an element can be determined by only considering hdvf's whose transcendence degree over k is bounded by n.

We compute the motivic conductor in many classical examples: For smooth group schemes and when restricted to trdeg 1, it coincides with the Rosenlicht–Serre conductor, for the group of Witt-vectors in positive characteristic it gives the Kato–Russell conductor, for integrable connections of rank 1 it coincides with the irregularity (up to shift), for finite characters of the fundamental group and lisse ℓ-adic sheaves of rank 1 it coincides with the Artin (or Brylinski–Kato–Matsuda) conductor. We also introduce the motivic conductor for torsors under finite commutative groups schemes over k and compute it in certain cases. On Kähler differentials in characteristic zero, it gives the pole order filtration. This last computation can be used to give a resolution-free characterization of rational singularities in characteristic zero. If time permits I explain how this computations can be used to compute certain tensor products of reciprocity sheaves (this last part is joint work with Sugiyama and Yamazaki).

Claude Sabbah • *What are irregular perverse sheaves?*

The talk will first explain the notion of Stokes-filtered local system as introduced by Deligne and generalized in higher dimension by Mochizuki, together with the Riemann–Hilbert correspondence for good meromorphic connections (over complex numbers). It will then focus on the recent approach of Kuwagaki, relying on the work of d'Agnolo and Kashiwara, which describes a category of irregular constructible complexes and of irregular perverse sheaves, and proves functorial properties and Riemann–Hilbert correspondence for arbitrary holonomic $\mathcal{D}$-modules (over complex numbers).

Takeshi Saito • CC

I will continue Beilinson's lecture to explain main results of the theory of characteristic cycle of étale sheaves and sketch the proofs.

Michael Temkin • *Reduction and lifting problems on Berkovich curves*

The classical theory based on stable reduction theorem associates to any nice curve X over an algebraically closed non-archimedean field K a metric subgraph called skeleton and a reduction curve over the residue field k. These objects are unique up to easily described modifications, and in fact they are the only invariants of tropical and k-algebraic nature one may associate to X.

One may wonder if an analogous reduction theory exists for more involved objects, such as morphisms of curves $f\colon Y \to X$ or a curve X with a differential form ω. It turns out that a straightforward approach based on simultaneous semistable reduction theorem or a naive reduction of ω is in general not satisfactory (though it works fine when f is tame). This happens because there exist additional tropical invariants, such as the different and residue functions.

In this talk, I will describe these combinatorial invariants, introduce associated reduction over k, and will explain how lifting theorems indicate that the new invariants form a complete set in the following two cases:

(1) a wild $f\colon Y \to X$ with local degrees not divisible by p^2 (joint with U. Brezner),

(2) a pair (X, ω) when the residue field k is of characteristic 0 (joint with I. Tyomkin).

Jean-Baptiste Teyssier • *Characteristic cycle and wild ramification for the nearby cycles of étale sheaves*

We will explain to which extent one can bound the wild ramification of the nearby cycles of an étale sheaf in terms of generic ramification invariants. This is joint work with Haoyu Hu.

Vaidehee Thatte • *Ramification theory for arbitrary valuation rings in positive residue characteristic*

In classical ramification theory, we consider extensions of complete discrete valuation rings with perfect residue fields. We would like to study arbitrary valuation rings with possibly imperfect residue fields and possibly non-discrete valuations of rank ≥ 1, since many interesting complications arise for such rings. In particular, the defect could be non-trivial (i.e. we can have a non-trivial extension, such that there is no extension of the residue field or the value group).

We present some new results for Artin–Schreier extensions of arbitrary valuation fields in positive characteristic p. These results relate the "higher ramification ideal" of the extension with the ideal generated by the inverses of Artin–Schreier generators via the norm map. We also introduce a generalization and further refinement of Kato's refined Swan conductor in this case. Similar results are true in mixed characteristic $(0, p)$.

Enlin Yang • *On the semi-continuity of characteristic cycles for étale sheaves*

In this talk, we show the generic constancy for singular supports and characteristic cycles for etale sheaves on a smooth fibration. Meanwhile, we show the failure of the lower semi-continuity of characteristic cycles in a higher relative dimension case, which is different from Deligne and Laumon's result in the relative curve case. This is joint work with Haoyu Hu.