

SOME REFERENCES ABOUT TORIC VARIETIES AS AMBIENT SPACES

On toric varieties

The bible (say the New testament) for toric varieties is the book of Cox, Little and Schenk [CLS11]. The Old testament is then Fulton's book [Ful93], but it is worth mentioning also some good Apocrypha available on M. Mustata's homepage [Mus] as well as Danilov's paper [Dan78]. The books by Oda [Oda78] and [Oda88] are also very interesting and cover some topics that are not addressed elsewhere.

About embeddability in toric varieties

The generalization of Kleiman's proof of Chevalley's conjecture to normal varieties is due to O. Benoist [Ben13]. See the references there about the earlier works.

The fundamental results about the existence of embeddings into toric (pre-)varieties are in Włodarczyk's paper [Wło93]. The refinements I talked about can be found in Hausen's paper [Hau02] and in the paper of Brenner and Schroer [BS03].

An introduction to toric prevarieties can be found in [ANH01]

A variant of the notion of ample families appear in the paper of A. Craw and D. Winn [CW13] where it is used to construct embeddings of Mori Dream Spaces in a special kind of toric varieties related to quiver representations. Nice topic.

Mori Dream Spaces

The seminal paper introducing Mori Dream Spaces (MDS) is due to Hu and Keel [HK00] and the fact that log-Fano varieties are MDS follows from the famous [BCHM10]. The most complete treatment of the subject (including neat embeddings) is certainly the book by I. Arzantzev, U. Derenthal, J. Hausen and A. Laface [ADHL14] but a shorter account can be found in the lecture notes of Hausen [Hau11] (close to what he gave in Lukecin last year). The Cox rings of del Pezzo surfaces were computed by Batyrev and Popov [BP04]. The construction with the blow-up of four planes in \mathbb{P}^5 is due to J. Levitt [Lev14].

Toric Mirror constructions and Reid's Fantasy

Reviews of the most common constructions of Calabi-Yau varieties as hypersurfaces or complete intersections in toric varieties can be found in D. Cox and S. Katz book [CK99] (which is also a good introduction to Mirror Symmetry in general) and in the survey of E. Clader and Y. Ruan [CR14]. The classification of the 4 dimensional reflexive polytopes has been performed by M. Kreuzer and H. Skarke [KS00] using their ad-hoc program PALP.

The original paper of Reid evoking the possibility of a connected web of Calabi-Yau 3-folds is [Rei87]. The physics literature about this subject is abundant but contains often inaccuracies. The case of complete intersections in products of projective spaces is the subject of a recent (math) preprint : [Wan16]. Mavlyutov has conjectured that it works for any complete intersections in toric varieties in [Mav05] and provided constructions allowing the deformation of toric varieties in [Mav09]. See also [IV12] for a variant of this in terms of T-varieties.

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